If the gain from one stage is insufficient and a multistage amplifier is used, then the bandwidth per stage will be related to the total bandwidth by the well-known relation

$$\Delta\omega_{\text{total}} = \Delta\omega_{\text{per stage}}\sqrt{2^{1/n} - 1} \tag{7}$$

in which n equals the number of stages.

For the second topic of this correspondence, we consider a narrow-band amplifier typically loaded by a resistive load, and the effect of the total output shunt capacitance, which limits the obtainable bandwidth. The total output shunt capacitance C_T is related to the output parallel susceptance across a two-port B by the well-known relation

$$C_T = \frac{1}{2}\frac{\partial B}{\partial \omega} \tag{8}$$

which, for a narrow-band amplifier, reduces to

$$C_T \cong \frac{1}{2}\frac{\Delta B}{\Delta \omega} \tag{9}$$

in which

$$\Delta B \cong B(\omega_0 + \tfrac{1}{2}\Delta\omega) - B(\omega_0 - \tfrac{1}{2}\Delta\omega). \tag{10}$$

A parallel load admittance $Y_l = G_l + jB_l$ can be expressed in terms of the reflection coefficient Γ_{ml} as

$$G_l + jB_l = \frac{1 - |\Gamma_{ml}|^2 + \Gamma_{ml}^* - \Gamma_{ml}}{|1 + \Gamma_{ml}|^2} \tag{11}$$

so that

$$jB_l = \frac{\Gamma_{ml}^* - \Gamma_{ml}}{|1 + \Gamma_{ml}|^2}. \tag{12}$$

Hence, from equations (9), (10), and (12)

$$C_T = \frac{1}{2\cdot\Delta\omega}\left|\frac{\Gamma_{ml}^*(\omega_0 + \frac{1}{2}\Delta\omega) - \Gamma_{ml}(\omega_0 + \frac{1}{2}\Delta\omega)}{|1 + \Gamma_{ml}(\omega_0 + \frac{1}{2}\Delta\omega)|^2} - \frac{\Gamma_{ml}^*(\omega_0 - \frac{1}{2}\Delta\omega) - \Gamma_{ml}(\omega_0 - \frac{1}{2}\Delta\omega)}{|1 + \Gamma_{ml}(\omega_0 - \frac{1}{2}\Delta\omega)|^2}\right|. \tag{13}$$

All terms on the right-hand side of (13) are either measurable or stated as a design criterion, and hence for effective constant resistive load matching the total output capacitance C_T, calculated as above for a given bandwidth requirement, must be greater than the active two-port output capacitance C_0.

Appendix

For a general n-port network with scattering matrix $[S]$, the energy dissipated, E_d, and the energy stored, E_s, are given respectively by the real and the imaginary parts of the product $[I_n^*]^T[V_n]$, in which * denotes conjugate and T denotes transpose. If a_k and b_k are the incident and reflected waves at the kth port, using the accepted definition for the n-port, $[b_n] = [S][a_n]$, it can be shown that

$$E_d = [a_n^*]^T[[U] - [S^*]^T[S]][a_n] \tag{14}$$

and

$$E_s = [a_n^*]^T[[S] - [S^*]][a_n] \tag{15}$$

in which $[U]$ is the unit matrix.

We define a matrix $[M]$ by the equation

$$[M] = [U] - [S^*]^T[S] \tag{16}$$

and a matrix $[N]$ by the equation

$$[N] = [S] - [S^*]^T. \tag{17}$$

Thus

$$E_d = [a_n^*]^T[M][a_n] \tag{18}$$

and

$$E_s = [a_n^*]^T[N][a_n]. \tag{19}$$

For time-invariant networks and sinusoidal excitation at the ports, the energy dissipated and the energy stored are both scalar quantities given by:

$$E_d = \sum_{\substack{j=1\\j=1}}^{n,n} a_i^* a_j M_{ij} \tag{20}$$

$$E_s = \sum_{\substack{j=1\\i=1}}^{n,n} a_i^* a_j N_{ij}. \tag{21}$$

And, further, if the incident waves to all ports have the same amplitude and phase, (20) and (21) reduce to:

$$E_d = |a|^2 \sum_{\substack{i=1\\j=1}}^{n,n} M_{ij} \tag{22}$$

$$E_s = |a|^2 \sum_{\substack{i=1\\j=1}}^{n,n} N_{ij}. \tag{23}$$

Hence, if Q for the n-port is defined as the ratio of stored energy to dissipated energy under the conditions of equal amplitude, in-phase incident waves, from (22) and (23),

$$Q_n = \frac{\sum_{\substack{i=1\\j=1}}^{n,n} N_{ij}}{\sum_{\substack{i=1\\j=1}}^{n,n} M_{ij}} \tag{24}$$

which, for a one-port, reduces to

$$Q = \frac{|S_{11} - S_{11}^*|}{1 - |S_{11}|^2}. \tag{25}$$

For the usual case, where the reference impedance is equal to the driving-point impedance, $S_{11} = \Gamma$, and

$$Q = \frac{|\Gamma - \Gamma^*|}{1 - |\Gamma|^2}$$

given earlier as (1).

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A Multioctave Microstrip 50-Ω Termination

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***Abstract*—A low-cost microstrip 50-Ω termination is described having a maximum VSWR of 1.46 (including the mismatch contributed by a microstrip launcher) from 1 to 18 GHz. It consists of a thin-film chip resistor with a matching structure. Experimental resistor characterization, equivalent-circuit modeling, and matching considerations are presented.**

This short paper describes an integrated-circuit 50-Ω termination, fabricated on 0.025-in ceramic microstrip, that operates from 1 to 18 GHz with good performance.

The termination is illustrated in Fig. 1. It consists of a thin-film

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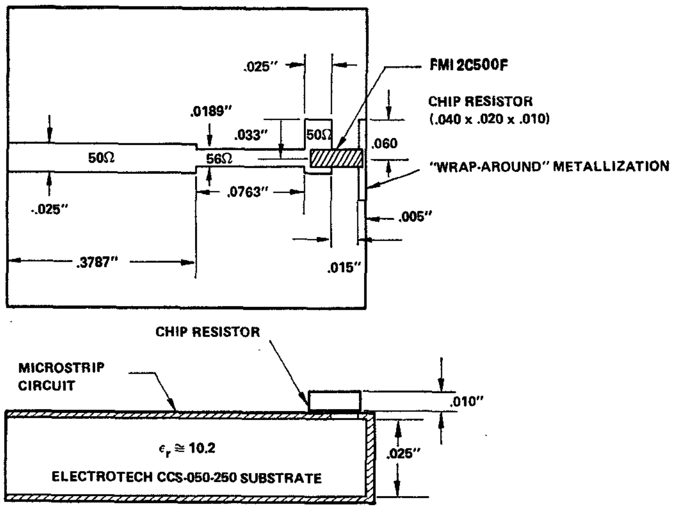

Fig. 1. Microstrip termination circuit.

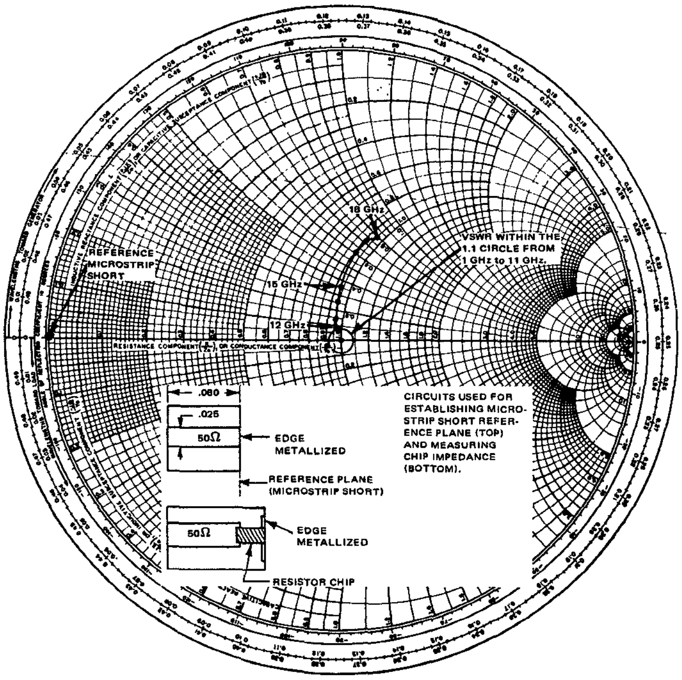

Fig. 2. Smith chart plot of resistor impedance (chip only).

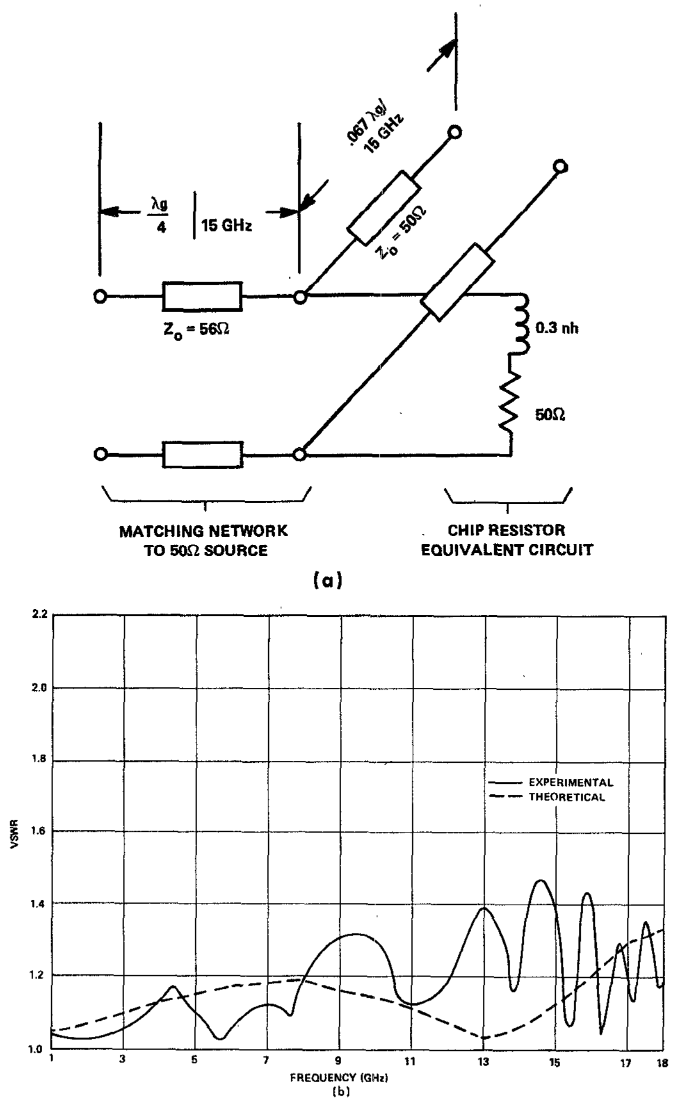

Fig. 3. (a) Termination equivalent circuit. (b) VSWR of microstrip termination.

chip resistor (manufactured by Film Microelectronics, Model no. 2C500F), RF grounded on one side by "wrap-around" metalization, and matched to a 50-Ω system with a stub/quarter-wavelength transformer combination. The resistor is fabricated by depositing a thin chrome layer on a 0.010-in thick ceramic chip, 0.020 in wide and 0.040 in long. Solder bumps are provided by the manufacturer at each end of the chip so that installation is a simple soldering task.

Experimental characterization of the chip was approached by establishing a microstrip reference short at the leading edge of the resistor (see Fig. 2). This was reasonably well accomplished in the same manner the chip is grounded; i.e., by using the short wrap-around metalization idea, which is merely the unetched metalization provided by the substrate supplier. Establishing this reference is not a trivial problem, particularly at *Ku* band, since the microstrip short becomes somewhat reactive. The technique employed was to re-establish the short every 500 MHz on an HP network analyzer before recording the approximate resistor impedance. Furthermore, to minimize dispersion effects, a very short section of microstrip, 0.080 in, was used from connector to reference plane. Fig. 2 shows the Smith chart experimental data, indicating that the chip is significantly inductive above about 12 GHz. Note that below 12 GHz a matching structure would not be required, as the chip reactance is very small in the 1- to 12-GHz range. A simple model for the resistor, an *RL* network, was quite adequate for describing the resistor chip throughout the 1- to 18-GHz range.

The matching circuit, necessary for *Ku*-band usage, is a 50-Ω open-circuited stub preceded by a 56-Ω quarter-wavelength transformer at 15 GHz, which brings the theoretical VSWR of the device within a 1.3 circle on the Smith chart over the 1- to 18-GHz frequency range. The resistor model and matching circuit are depicted in Fig. 3(a).

The termination was tested, using precision reflectometers and a reasonably well-matched 3-mm-to-microstrip launcher developed at Applied Technology. Experimental results are shown in Fig. 3(b) and compared with theoretical predictions. The maximum VSWR from 1 to 18 GHz of 1.46 occurred at about 14.5 GHz. Note that these data include the discontinuity from a microstrip launcher, accounting for some of the periodic fluctuations in measured VSWR. If the connector were eliminated from the measurement, as was done for chip characterization, a closer correlation with the theoretical model could have been anticipated.

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